Calculus
Measuring Speed

Name: Oliver

(1) Times and positions of a bike rider are given in the table below.

Time (Seconds)	0	1	2	3	4	5	6
Position (Feet)	0	2.8	7.2	15.3	25.6	38.7	46.1

(a) Compute the average speed (in feet/second) of the rider during each of the following time intervals.

[1, 2] [2, 3] [3, 4] [4, 5] [5, 6]

$\frac{7.2-2.8}{1} = 4.4$

$\frac{15.3-7.2}{1} = 8.1$

10.3

13.1

7.4

(b) We want to approximate the rider's instantaneous speed at time $t = 2$ seconds. Compute the average speed of the rider during each of the following time intervals.

[2, 5] [2, 4] [2, 3]

$\frac{38.7-7.2}{3} = 10.5$

$\frac{25.6-7.2}{2} = 9.2$

8.1

(c) Which of the approximations from (b) is probably best? Why?

interval [2, 3]
since it's the smallest the interval

(d) Compute the average speed of the rider during the time interval [1, 3].

$\frac{15.3-2.8}{2} = 6.25$

(e) How do you think this approximation compares to the approximation over the interval [2, 3]?

not sure...

(f) What information would you need to find a better estimate of the rider's instantaneous speed at $t = 2$ seconds?

more speed info / position info close to $t = 2$ sec

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(2) Now, consider a simple model of the rider's position: $s(t) = t^2 + 3t - 1$. Use this model for the computations below.

(a) Compute the average velocity of the rider over each of the following intervals.

[2, 4] [2, 3] [2, 2.5]

$\frac{27-9}{2}$ $= 9$

$\frac{17-9}{1} = 8$

$\frac{12.75-9}{.5} = 7.5$

(b) Any opinion about which of these approximations is closest to the rider's instantaneous speed at $t = 2$?

[2, 2.5]

(c) Compute a better estimate for the rider's instantaneous speed at $t = 2$ seconds. Show your method.

[2, 2.1] ⇒ 7.1

(d) Compute an even better estimate for the rider's instantaneous speed at $t = 2$ seconds. Show your method.

[2, 2.01] ⇒ 7.01

(e) Explain how the expression $\dfrac{s(2+h)-s(2)}{h}$, with various values of h, gives approximations for the rider's speed at $t = 2$ seconds. Simplify this expression for the given model for position $s(t)$.

avg speed on [2, 2+h]

$$\frac{((2+h)^2 + 3(2+h) - 1) - (9)}{h} = \frac{4 + 4h + h^2 + 6 + 3h - 1 - 9}{h}$$

$$= \frac{7h + h^2}{h} = 7 + h$$

(f) What happens to this expression as h approaches 0?

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