Calculus
Introduction to L'Hopital's Rule

Name: Dover

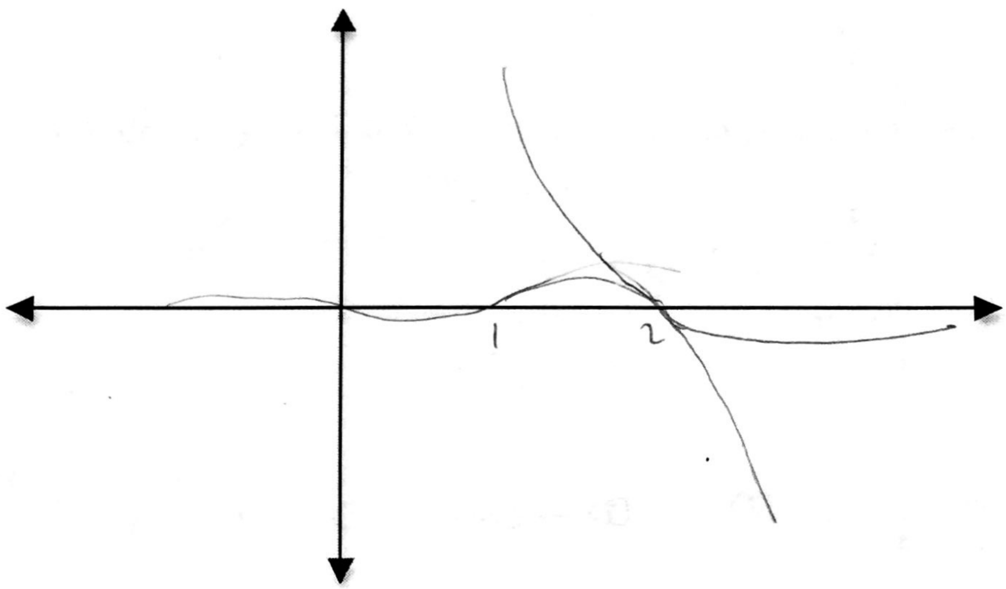
We want to find $\lim\limits_{x\to 2} \dfrac{(x-5)\cdot\ln(x-1)}{\left(x-x^3\right)\cdot\sin(x-2)}$.

First, check to see what happens if we substitute $x = 2$ into the expression.

$\frac{0}{0}$

Estimate the limit on your calculator by substituting values of x near $x = 2$.

Obviously, some of our earlier algebraic techniques won't get us very far in this problem.
Let $f(x)=(x-5)\cdot\ln(x-1)$ and let $g(x)=\left(x-x^3\right)\cdot\sin(x-2)$. Plot both functions together.



Now zoom in around $x = 2$ so that both functions look (more or less) like straight lines.

Write the equation of the tangent line to f at $x = 2$.

$f'(2) = -3 \rightarrow \quad y - 0 = -3(x-2)$

Write the equation of the tangent line to g at $x = 2$.

$g'(2) = -6 \qquad y - 0 = -6(x-2)$

Since these linear functions are good approximations of f and g near $x = 2$, we may substitute

Find $\lim\limits_{x\to 2} \dfrac{\text{tangent line to } f}{\text{tangent line to } g}$. $= \lim \dfrac{-3(x-2)}{-6(x-2)} = \dfrac{1}{2}$

assume $\lim_{x \to a} f(x) = 0 = \lim_{x \to a} g(x)$

$$\lim_{x \to a} \frac{f(x)}{g(x)} \overset{\text{[illegible]}}{\Rightarrow} \frac{f'(a)}{g'(a)} = \lim_{x \to a} \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}} = \lim_{x \to a} \frac{f(x)-f(a)}{g(x)-g(a)}$$

$$= \lim_{x \to a} \frac{f(x)}{g(x)}$$

AP stuff = [illegible]

handout on scoring

students [illegible]

Graphs $\begin{cases} y = x \\ y = \sin x \end{cases}$ $\quad$ $y = x$, $y = \sin(2x)$

$\boxed{\frac{0}{0}, \frac{\infty}{\infty}}$, $0 \cdot \infty$, $\infty - \infty$, 0^0, ∞^0, 1^∞