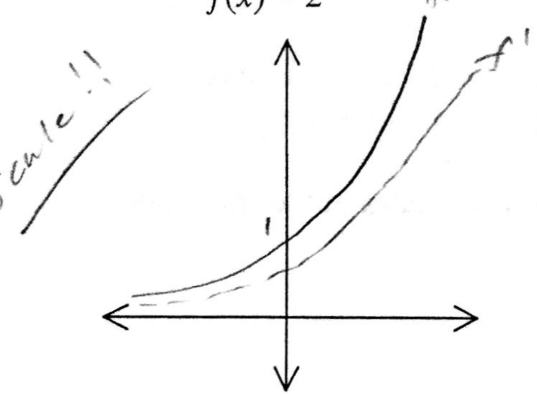
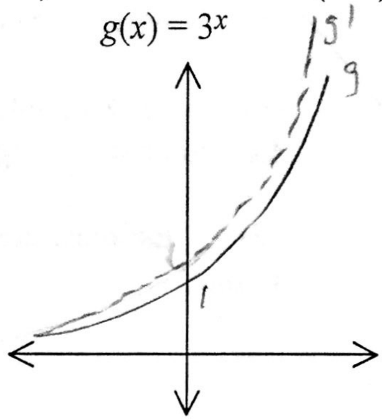
Derivatives of Exponential Functions Name:__________________________

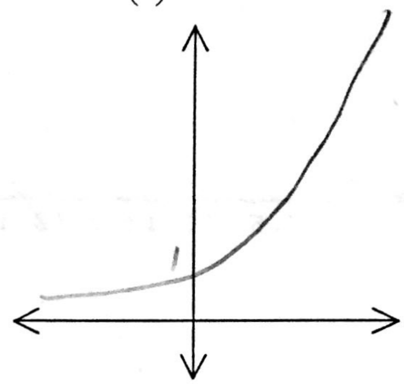
This activity sheet is designed to find the derivative of the function $f(x) = b^x$.

(1) Graph each function (——) and its derivative (- - -). Try the window $[-2, 4] \times [-2, 20]$.

$f(x) = 2^x$ $g(x) = 3^x$ $h(x) = e^x$







(2) What can be said about h and h'?

Same!

(3) To investigate the derivative of $f(x) = b^x$, we consider the ratio of the derivative to the function itself. In other words, we want to consider the set of ordered pairs of the form (b, ratio of y'/y) for a variety of values of b.

Begin with the specific case where $b = 2$. On the "y=" screen, let :

(TI-89) y1=2^x, y2 = nDeriv(y1(x), x), and let y3 = y2(x)/y1(x) OR

(TI-83) $Y_1 = 2$^X, Y_2 = nDeriv(Y_1, X, X) and $Y_3 = Y_2/Y_1$

Use "Table." What is true about $y3$? What is its value?

Change $y1$ and repeat this process for each of the values of b given below. Fill in the ordered pairs with $y3$ = ratio of y'/y. Plot these ordered pairs on the graph.

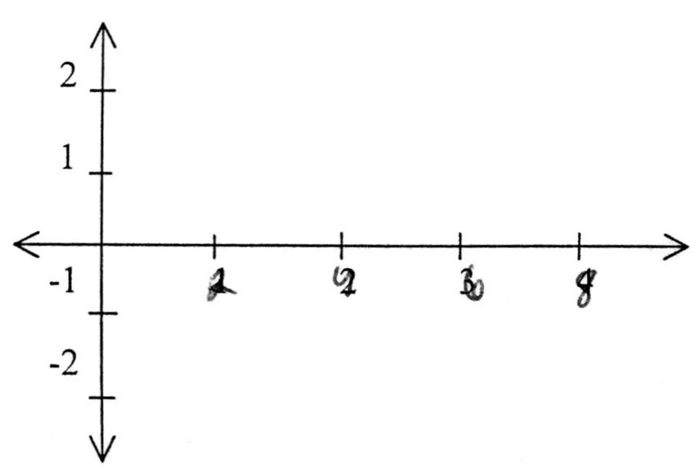
(2, .693), (.25, −1.386)

(.5, −.693), (1.5, .405)

(3, 1.0986), (4, 1.3863)

(5, 1.6094), (6, 1.7918)

(7, 1.9459), (8, 2.0794)



What does this function look like?

log!

In short, we have the relationship $(b,$ ___ln b___$)$. This means that the ratio

of $\frac{\frac{d}{dx}(b^x)}{b^x} =$ ___ln b___, so the derivative of $f(x) = b^x$ is $\frac{d}{dx}(b^x) =$ ___$b^x \ln b$___.

(4) We've seen that the derivative of $y = b^x$ is a multiple of the function itself. Show this by setting up the derivative by definition and factoring out b^x. The remaining

expression may be seen as the derivative of ___b^x___ at $a =$ ___0___.

$$y' = \lim_{h\to 0} \frac{b^{x+h} - b^x}{h} = \lim_{h\to 0} b^x \cdot \left(\frac{b^h - 1}{h}\right) = b^x \cdot \lim_{h\to 0} \frac{b^h - 1}{h}$$

(5) What does your derivative rule imply about the derivative of $y = e^x$? Show.

$$y' = e^x \ln e = e^x$$

(6) Find the derivative of each of the following using the rule found above. (For some, you will want to rewrite the expression using properties of exponents.)

$y = 10^x$

$y' = 10^x \ln 10$

$y = 4 \cdot 3^x$

$y' = 4 \cdot 3^x \ln 3$

$y = 4^{2x} = 16^x$

$y' = 16^x \ln 16$

$y = (1/2)^x$

$y' = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right)$

$y = \frac{3}{5^x} = 3 \cdot \left(\frac{1}{5}\right)^x$

$y' = 3 \cdot \left(\frac{1}{5}\right)^x \ln\left(\frac{1}{5}\right)$

$y = x^2 + 2^x + 2$

$y' = x^2 + 2^x \ln 2$

$y = 3 \cdot 6^{-x} = 3\left(\frac{1}{6}\right)^x$

$y' = 3 \cdot \left(\frac{1}{6}\right)^x \cdot \ln\left(\frac{1}{6}\right)$

$y = e^{2x} + x^{2e} = (e^2)^x + x^{2e}$

$y' = (e^2)^x \cdot \ln(e^2) + 2e\, x^{2e-1}$

$= 2e^{2x} + 2e\, x^{2e-1}$

$y = 3e^{x+2} + 5^{x-1}$

$y' = 3e^{x+2} + 5^{x-1} \cdot \ln 5$