Calculus Coffee Name: Dover

Everything you wanted to know about coffee...

Newton's Law of Cooling states that the rate of change of the temperature of the coffee is proportional to the difference between the temperature of the coffee and the room (surrounding) temperature.

(1) If y is the temperature of the coffee and T is the room temperature, express this as a differential equation.

$$y' = k(y - T)$$

(2) Solve the DE in terms of constants k and T.

$$\frac{dy}{dt} = k(y - T)$$

$$\frac{dy}{y - T} = k\,dt$$

$$\ln|y - T| = kt + C$$

$$|y - T| = e^{kt+c}$$

$$|y - T| = e^{c} \cdot e^{kt}$$

$$y - T = (\pm e^{c}) \cdot e^{kt}$$

$$y = Ae^{kt} + T$$

$$\boxed{y = T + Ae^{kt}}$$

(3) Assume that the room temperature is 70° F and the coffee is initially 190°F. After 3 minutes, the coffee is 150°F. Find the specific equation for the temperature of the coffee at any time t, where t is in minutes.

$\underline{t = 0}$

$$190 = 70 + A$$

$$A = 120$$

$\underline{t = 3}$

$$150 = 70 + 120e^{k \cdot 3}$$

$$\frac{8}{12} = e^{3k}$$

$$k = -.135$$

$$y = 70 + 120e^{-.135t}$$

(4) Write the differential equation using the specific values found for k and T.

$y' = -.135(y - 70)$

Draw the slope field for the differential equation on your calculator. Draw a rough version below. (What's a good window?) Then graph your solution function (from 3) on your calculator to make sure it seems to be correct. (Does it match the slope field?) Sketch it below as well.

(5) What if the value of k had been $-.05$? Would the coffee cool more quickly or more slowly? What might have accounted for this different value?

more slowly since $|\text{rate}|$ is closer to 0

insulation of cup

(6) Assume now that I just took a can of soda from the refrigerator. Would the differential equation look the same? Does anything change? Sketch a graph of a typical solution curve. Be sure that your curve agrees with the slope field.

yep! same DE