Calculus
Approximations 1 – Distance

Name: Dover

Example 1: A car is speeding up during a time period from $t = 0$ to $t = 4$ seconds. Values of the velocity are given in the table below for selected values of t.

Time (seconds)	0	1	2	3	4
Velocity (feet/sec)	12	20	28	36	40

(1) Estimate the distance traveled by the car during these 4 seconds. (Show your method.)

(options!)

(2) Find an underestimate for the distance the car traveled. (Show your method.)

$1 \cdot 12 + 1 \cdot 20 + 1 \cdot 28 + 1 \cdot 36 = 96$

(3) Find an overestimate for the distance the car traveled. (Show your method.)

$1 \cdot 20 + 1 \cdot 28 + 1 \cdot 36 + 1 \cdot 40 = 124$

Now, more data is actually known.

Time	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Velocity	12	15	20	23	28	34	36	39	40

(4) Using this data, find an underestimate for the distance the car traveled.

$.5 \cdot (12 + 15 + 20 + 23 + 28 + 34 + 36 + 39)$

$= 105$

(5) Using this data, find an overestimate for the distance the car traveled.

$.5 (15 + 20 + 23 + 28 + 34 + 36 + 39 + 40)$

$= 117.5$

(6) How do your answers to (2) and (3) compare to your answers from (4) and (5)? Explain.

both answers in (4) + (5) are closer to the average

Example 2: Water is leaking out of a container. It leaks out quickly at first and then slows with time. The rate of leakage, given by the number of liters lost per minute, is given for selected values of time t in the table below.

time (minutes)	0	2	5	6
liters per minute	5.3	4.1	2.7	1.8

(1) Using the data in the table, find an underestimate for the amount of water that may have leaked out of the container during the 6 minutes. (Show your method.)

$2 \cdot 4.1 + 3 \cdot 2.7 + 1 \cdot 1.8$

$= 18.1$

(2) Using the data in the table, find an overestimate for the amount of water that may have leaked out of the container during the 6 minutes. (Show your method.)

$2 \cdot 5.3 + 3 \cdot 4.1 + 1 \cdot 2.7$

$= 25.6$

(3) What would be necessary to make better estimates?

more intervals (at more times)

Quick Summary:

(1) How do your under- and over-estimates change if you have more data?

They get closer together

(2) How did your method for finding the under- and overestimates change from the distance problem to the water problem? What about the data caused this to change?

(3) On the first problem, obviously, an underestimate for the distance traveled would have been 0 feet, even if that wouldn't have been particularly useful. If the velocity had both increased and decreased during those four seconds, would you have been able to find more useful under- and overestimates? Explain.