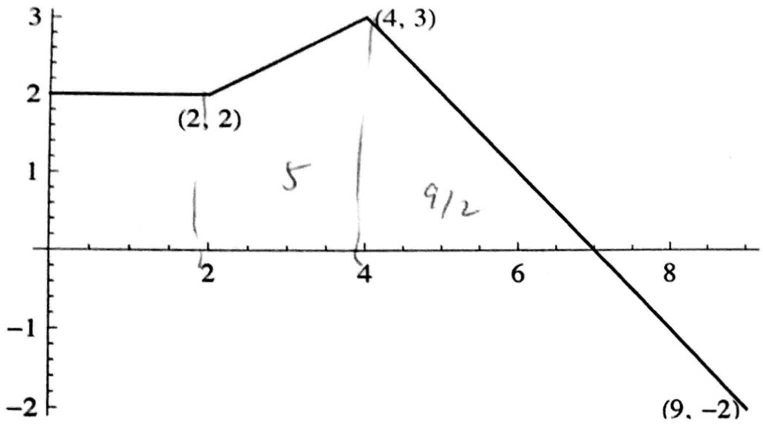
FTC Problems

Name:________________________

(1) Let f be the function graphed below and let $g(x) = \int_2^x f(t)\,dt$. (Notice the lower limit of integration on g.)



Evaluate each of the following.

(a) $g(0)$ $= -4$

(b) $g(2)$ $= 0$

(c) $g(4)$ $= \frac{1}{2}(2+3)\cdot 2$

$= 5$

(d) $g(9)$ $= 5 + \frac{9}{2} - 2 = \frac{15}{2}$

(e) $g'(4)$

$= f(4) = 3$

(f) $g'(7)$ $= 0$

(g) What is the maximum of g on [0, 9]? Justify.

Consider $x = 0, 7, 9$

$g(0) < 0$, $g(7) = 5 + \frac{9}{2} = \frac{19}{2}$, $g(9) = \frac{15}{2}$ (above)

$\therefore$ max $= \frac{19}{2}$ at $x = 7$

(h) What is the minimum of g on [0, 9]? Justify.

Consider $x = 0, 9$

$g(0) = -4$, $g(9) = \frac{15}{2}$

min $= -4$ at $x = 0$

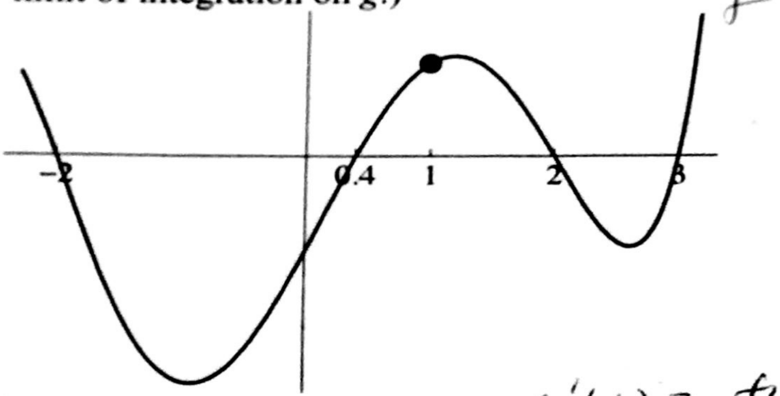
(i) Where is g concave down? Justify.

$g' = f$ decr $\Rightarrow (4, 9)$

(j) Write the equation of the tangent line to g at $x = 4$.

$m = g'(4) = 3$, $g(4) = 5$ $\Rightarrow y - 5 = 3(x - 4)$

(2) Let f be the function graphed below and let $g(x)=\int_1^x f(t)\,dt$. (Notice the lower limit of integration on g.)



In (a) – (f), determine whether each of the following is positive, negative, or zero. Justify.

(a) g(1) $= 0$

(b) g′(1) > 0 since $g'(1) = f(1) > 0$ by graph

(c) g(2) > 0 accumulating area from 1 to 2

(d) g′(2) $= 0$ since $g'(2) = f(2) = 0$ (by graph)

(e) g(3) > 0 accumulating area from 1 to 2, then losing some from 2 to 3 but very little

(f) g(−2) > 0 going backwards, from $x=1$, losing a little to .4, then adds much more going back to -2 w/ $f<0$

(g) What is the x-coordinate of the maximum of g on [−2.3, 3.2]? Justify.

$x = -2$

a bigger net result than going from 1 to 2

(h) For what x (approximate) is g concave down? Justify.

$g' = f$ decr

$(-2.3, -1)$, $(1.2, 2.5)$