Calculus Name:_____

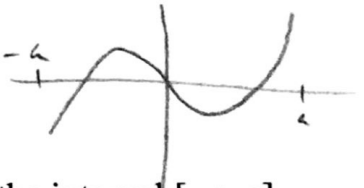
Properties of Definite Integrals

In each problem below, make (good!) conjectures about properties of integrals, based on your experience with mathematics.

(1) $\int_a^a f(x)dx =$ 0 Why?

(2) If $f$ is odd, then intervals of the form $[-a, a]$ are interesting. State a property about the integral of an odd function $f$.

$\int_{-a}^{a} f(x)\,dx = 0$




If $g$ is even, state a property about the integral of $g$ on the interval $[-a, a]$.

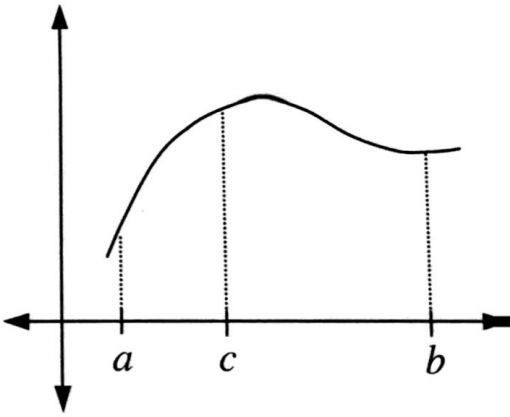
$\int_{-a}^{a} g(x)\,dx = 2\int_{0}^{a} g(x)\,dx$

(3) $\int_b^a f(x)dx =$ $-\int_a^b f(x)\,dx$ (What happens when you go backwards in mathematics?) Is this only true for $a < b$? Why?

(4) Consider the drawing to the right. This suggests:

$\int_a^b f(x)dx =$ $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Is the order of $a$, $b$, and $c$ important?




(5) $\int_a^b k \cdot f(x)dx =$ _____, where $k$ is a constant.

Draw a picture to illustrate this.

(6) Factoid: $\int_0^{\pi} \sin x\,dx = 2$ Find each of the following.

$\int_0^{\pi} 3\sin x\,dx$ $\int_0^{3\pi} 4\sin x\,dx$

6 8

(7) $\int_a^b (f(x)-g(x))dx =$ _____

Draw a picture to illustrate this. Explain what this represents if $f(x) \geq g(x)$.

area between f + g

$\int_a^b (f(x)+g(x))dx =$ _____

Draw a picture to illustrate this. Explain what this represents.

ex $\int_0^{\pi/2} (\cos x +$

Do these properties change if $f$ and/or $g$ is negative? Explain.

no

(8) Find each of the following.

$\int_0^{\pi} (\sin x + 2)dx$ $\int_0^{\pi/2} (3\cos x - 2)dx$

$2 + 2\pi$ $3 - 2 \cdot \pi/2 = 3 - \pi$