Assume: (1) $\int_a^b f(x)\,dx$ = notation for signed area

(2) basic antiderivatives

Calculus   Accumulating Area   Name: Oren

(1) Let $A(x) = \int_0^x 4\,dt$.

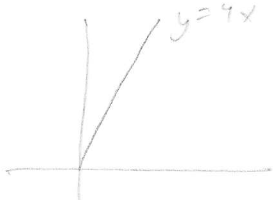
(a) Sketch a graph of $f(t) = 4$.

(b) Use geometry to fill in the chart below and generalize it for an arbitrary value of x.

x	0	1	2	3	…	x
$A(x)$	0	4	8	12	…	$4x$

(c) Sketch the graph of A. What is the slope? The y-intercept?

4   0



(d) How would we change the definition of $A(x)$ that is given above to create a new function $B(x)$ with slope 5?

$B(x) = \int_0^x 5\,dt$

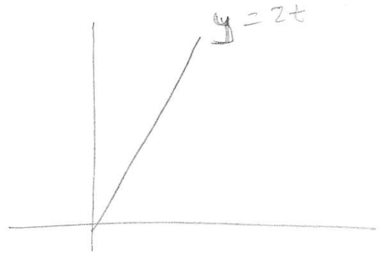
(e) If we define $C(x) = \int_{-1}^{x} 4\,dt$, how does the graph of C relate to the graph of A from part (c) above?

$C = A + 4$

want vertical shift

(2) Let $F(x) = \int_0^x 2t\,dt$.

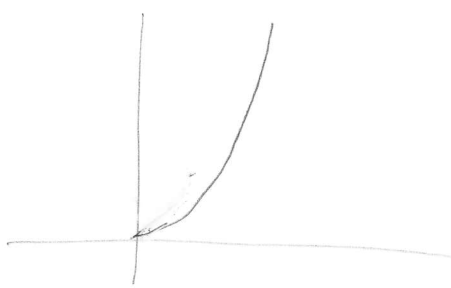
(a) Sketch a graph of $f(t) = 2t$ for $t \geq 0$.



(b) Use geometry to fill in the chart below and generalize it for an arbitrary value of x.

x	0	1	2	3	4	…	x
$F(x)$	0	1	4	9	16	…	x^2

(c) Sketch the graph of F for $x \geq 0$. Then use the graph of $f(t) = 2t$ to explain why the graph of F does not have a constant slope (as on the first page).

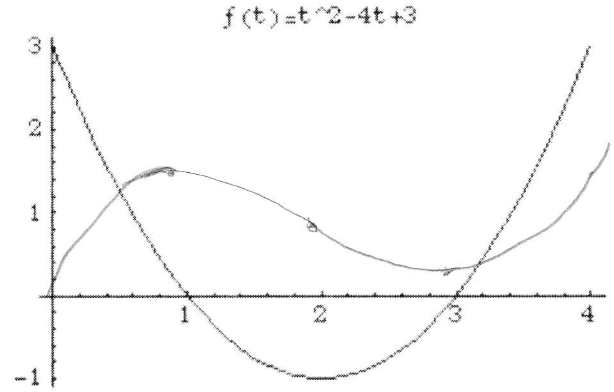


more + more area added as t increases

(d) Estimate the slopes of F at $x = 1, 2, 3$, and 4. (Zoom in or do something on your calculator!) What is the relationship between F and the function $f(t) = 2t$?

$F' = f$

(3) Let $G(x)=\int_0^x \left(t^2-4t+3\right)dt$.



(a) Fill out the chart below, using geometry to make some good approximations.

x	0	1	2	3	4
$G(x)$	0	1.4	.8	.2	1.6

(b) On the graph above, make a rough sketch of the graph of G by using the graph of $y = f(t)$.

(c) Consider the function $h(t)=\dfrac{t^3}{3}-2t^2+3t$. How does this function h relate to your graph of G?

$h = g$

(d) Let $K(x)=\int_1^x f(t)dt$. How does G relate to K? Explain.

K is less than G

$G(x) - K(x) = \underbrace{\int_0^1 f(t)dt}_{\text{constant}}$

(4) (a) For any continuous function $y = f(t)$, make a conjecture which relates A and f, where $A(x) = \int_a^x f(t)\,dt$.

$A' = f$

(b) Expand your conjecture to explain the effect of the choice of a on the function A.

The "a" will affect the vertical shift

$A(a) = 0$

(5) Use your conjecture to:

(a) find $A(x)$ if $A(x) = \int_0^x (t^3 - 4t + 1)\,dt$

$= \frac{x^4}{4} - 2x^2 + x$

(b) find $B(x)$ if $B(x) = \int_1^x (t^3 - 4t + 1)\,dt$.

$B(x) = \frac{x^4}{4} - 2x^2 + x + C$

$B(1) = 0 \Rightarrow 0 = \frac{1}{4} - 2 + 1 + C$

$\Rightarrow C = \frac{3}{4}$

$B(x) = \frac{x^4}{4} - 2x^2 + x + \frac{3}{4}$

(6) (a) Find $A(x)=\int_0^x \left(6t^2+3\right)dt$

$= 2x^3 + 3x$

(b) Find $A(5)=\int_0^5 \left(6t^2+3\right)dt$.

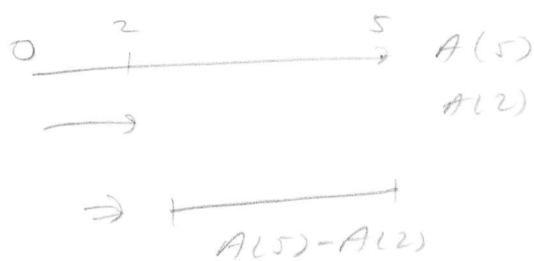
$= 2\cdot 5^3 + 3\cdot 5 = 265$

(c) Find $A(2)=\int_0^2 \left(6t^2+3\right)dt$.

$= 2\cdot 2^3 + 3\cdot 2 = 22$

(d) Find $\int_2^5 \left(6t^2+3\right)dt$ and explain this geometrically in terms of the integrals in parts (b) and (c).

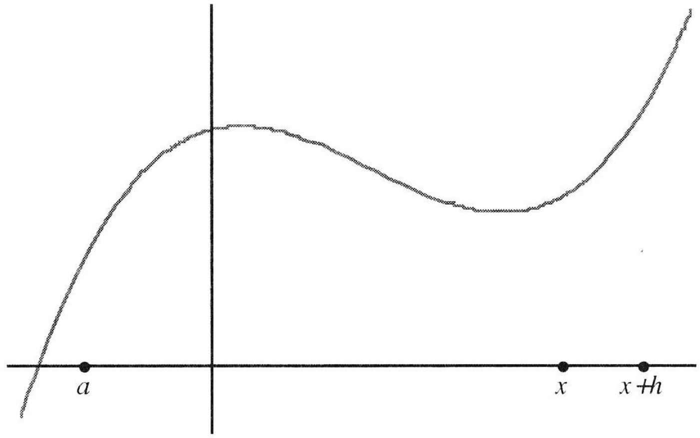
$= 243$



(7) Generalize: if F is an antiderivative of f, find $\int_a^b f(t)\,dt$.

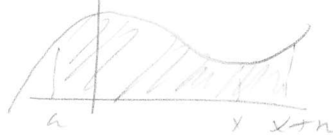
$F(b)-F(a)$

(8) If $A(x) = \int_a^x f(t)\,dt$, interpret each of the following expressions geometrically in terms of the picture to the right.

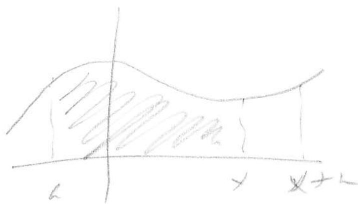


$y = f(t)$

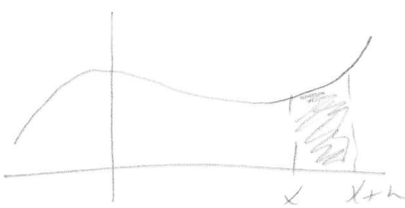
(a) $A(x+h)$



(b) $A(x)$



(c) $A(x+h) - A(x)$



(d) $\dfrac{A(x+h) - A(x)}{h}$

f_{avg}

note:
avg value stuff
$A(\text{rect}) = A(\text{under } f)$
$f_{avg} \cdot h = A(x+h) - A(x)$
$f_{avg} = \dfrac{A(x+h) - A(x)}{h}$

(e) What happens to the expression in part (d) as $h \to 0$?

Compute!