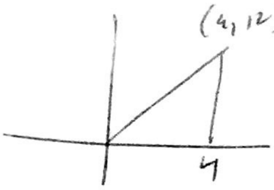
Calculus
Properties 2 – Practice

Name:________________________

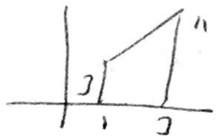
Find the value of each of the following integrals by using the geometry of the regions and properties of integrals.

(1) $\int_0^4 3x\,dx$



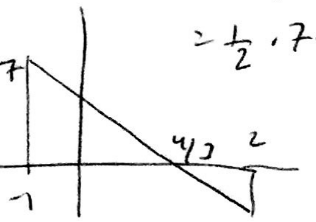
$A = \frac{1}{2}\cdot 4\cdot 12$
$= 24$

(2) $\int_1^3 (4x-1)\,dx$



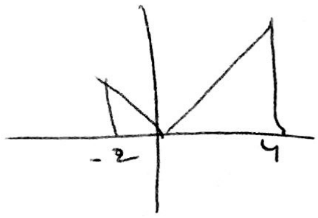
$\frac{1}{2}\cdot 2\cdot(3+11)$
$= 14$

(3) $\int_{-1}^2 (4-3x)\,dx$



$= \frac{1}{2}\cdot 7\cdot \frac{7}{3} - \frac{1}{2}\cdot\frac{2}{3}\cdot 2$
$= \frac{49}{6} - \frac{2}{3}$
$= \frac{45}{6} = \frac{15}{2}$

(4) $\int_{-2}^4 |x|\,dx$



$= 2 + \frac{1}{2}\cdot 4\cdot 4$
$= 10$

(5) $\int_{-2}^{-2} (4x^2-3x+1)\,dx$

0

(6) $\int_{-2}^2 (2x^3-3x)\,dx$

0 (odd)

(7) If $\int_{-1}^3 f(x)\,dx = 6$, $\int_3^5 f(x)\,dx = -2$, and $\int_3^8 f(x)\,dx = 5$, find each of the following.

(a) $\int_{-1}^5 f(x)\,dx$

$= 6 - 2 = 4$

(b) $\int_5^8 f(x)\,dx$

$= \int_3^8 f(x)\,dx - \int_3^5 f(x)\,dx$
$= 5 - (-2) = 7$

(c) $\int_{-1}^3 (2f(x)+4)\,dx$

$2\cdot 6 + 4\cdot 4$
$= 12 + 16 = 28$

(d) $\int_3^5 (3 - f(x))\,dx$

$= 3\cdot 2 - (-2) = 8$

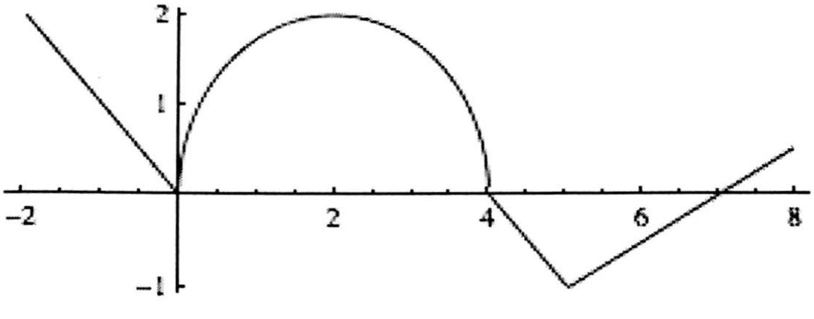
[illegible]

(e) Find the average value of f on [3, 5].

$\frac{1}{2}\int_3^5 f(x)\,dx = \frac{1}{2}(-2) = -1$

Name:____________________________

(8) The graph of $y = f(t)$, comprised of a semi-circle and line segments, is shown below. Let $g(x) = \int_0^x f(t)\,dt$.



Evaluate each of the following.

(a) $g(0)$ $= 0$

(b) $g(2)$ $= \frac{1}{4}\pi \cdot 2^2 = \pi$

(c) $g(-2)$

$= -2$

(d) $g(5)$

$= 2\pi - \frac{1}{2}$

(e) $g(7)$

$2\pi - \frac{1}{2} - 1$

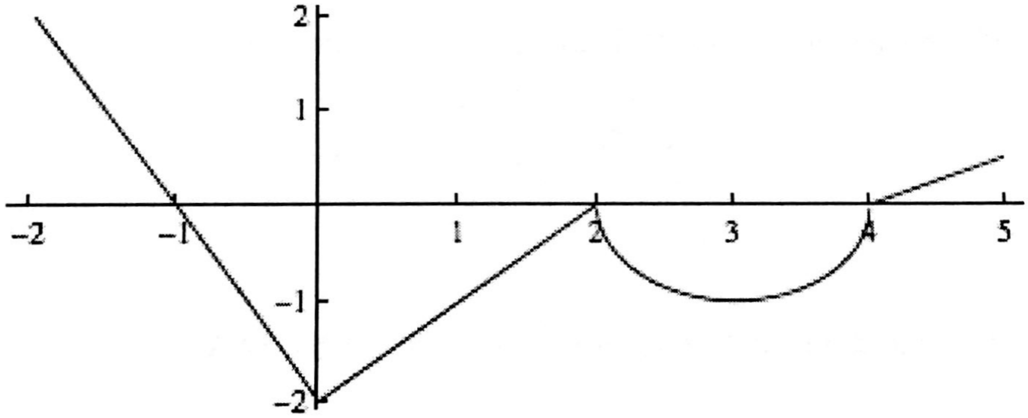
$= 2\pi - 3/2$

(f) $g(8)$

$2\pi - \frac{1}{2} - 1 + \frac{1}{4}$

$= 2\pi - \frac{5}{4}$

(9) The graph of $y = h(t)$, comprised of a semi-circle and line segments, is shown below. Let $k(x) = \int_2^x h(t)\,dt$.



Evaluate each of the following.

(a) $k(2)$ $= 0$

(b) $k(0)$ $= 2$

(c) $k(-1)$

$= -(2-1)$

$= +3$

(d) $k(-2)$

$= -(2-1+1)$

$= +2$

(e) $k(4)$

$= -\frac{1}{2}\pi$

(f) $k(5)$

$= -\frac{\pi}{2} + \frac{1}{4}$