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Limits Involving Infinity

Let $f(x) = \dfrac{4x^2 - 2x}{x^2 - x - 6}$. $= \dfrac{2x(2x-1)}{(x-3)(x+2)}$

How do we find vertical asymptotes of a function? What are the vertical asymptotes of *f*?

Search for zeroes in denominator, make sure num $\neq 0$.

$x = -2, 3$

Investigate the behavior of *f* around these asymptotes by find each of the following limits.

$\lim\limits_{x \to -2^-} f(x) =$ $+\infty$ $\qquad$ $\lim\limits_{x \to 3^-} f(x) =$ $-\infty$

$\lim\limits_{x \to -2^+} f(x) =$ $-\infty$ $\qquad$ $\lim\limits_{x \to 3^+} f(x) =$ $+\infty$

How do we find horizontal asymptotes of a function?

See what happens at $x \to +\infty$ or $-\infty$

Guess: The horizontal asymptote of *f* is $y=4$.

Graph the function to confirm your limits and your guess for the asymptote. (You may wish to copy the graph above, to the right.)

For each of the following, think about what happens to the fraction as *x* grows larger and guess the value of each limit. Note that this is another way of asking you to find *k* where $y = k$ is the horizontal asymptote. Graph each function to check your guess.
if there is horiz asymptote

(1) $\lim\limits_{x \to \infty} \dfrac{2x+1}{x} = 2$ $\qquad$ (2) $\lim\limits_{x \to \infty} \dfrac{4x^2 - 1}{3x^2 + 2x} = \dfrac{4}{3}$

(3) $\lim\limits_{x \to \infty} \dfrac{5x^3}{2x^2 + 1} = +\infty$ $\qquad$ (4) $\lim\limits_{x \to \infty} \dfrac{2 - 4x}{x^3 + 5x} = 0$

To approach this more methodically, consider first $y = \dfrac{1}{x^n}$ where $n \geq 1$.

What happens to *y* as $x \to +\infty$? $y \to 0$

What happens to *y* as $x \to -\infty$? $y \to 0$

Return to $f(x)=\dfrac{4x^2-2x}{x^2-x-6}$. Again, algebra will help to change the form of the function.

Multiply both the numerator and denominator by $\dfrac{1}{x^2}$ since x^2 is the largest power of x in the denominator. We have $\lim\limits_{x\to\infty}\dfrac{4x^2-2x}{x^2-x-6}\cdot\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}}=\lim\limits_{x\to\infty}\dfrac{4-\frac{2}{x}}{1-\frac{1}{x}-\frac{6}{x^2}}=4$

since the terms $\dfrac{2}{x}$, $\dfrac{1}{x}$, and $\dfrac{6}{x^2}$ all approach 0 as x approaches ∞.

Use the above method to determine the following limits. (Show your work!)

(1) $\lim\limits_{x\to\infty}\dfrac{4x^3-5}{x^3+2x^2}\cdot\dfrac{\frac{1}{x^3}}{\frac{1}{x^3}}=\lim\dfrac{4-5/x^3}{1+\frac{2}{x}}=4$

(2) $\lim\limits_{x\to\infty}\dfrac{4x^4-6x+1}{1-2x^4}\cdot\dfrac{\frac{1}{x^4}}{\frac{1}{x^4}}=\lim\dfrac{4-\frac{6}{x^3}+\frac{1}{x^4}}{\frac{1}{x^4}-2}=-2$

(3) $\lim\limits_{x\to\infty}\dfrac{4x^8-6x^3+5x}{x^{10}+1}\cdot\dfrac{\frac{1}{x^{10}}}{\frac{1}{x^{10}}}=\lim\dfrac{\frac{4}{x^2}-\frac{6}{x^7}+\frac{5}{x^9}}{1+\frac{1}{x^{10}}}=0$

(4) $\lim\limits_{x\to-\infty}\dfrac{x^4-3x+1}{3x^3+6x}\cdot\dfrac{\frac{1}{x^3}}{\frac{1}{x^3}}=\lim\dfrac{x-\frac{3}{x^2}+\frac{1}{x^3}}{3+\frac{6}{x^2}}=-\infty$

Assorted: Determine each of the following limits.

(5a) $\lim\limits_{x\to2^-}\dfrac{x+2}{x-2}=-\infty$ (b) $\lim\limits_{x\to2^+}\dfrac{x+2}{x-2}=+\infty$ (c) $\lim\limits_{x\to2}\dfrac{x+2}{x-2}=$ DNE

(6) $\lim\limits_{x\to-1^-}\dfrac{x-2}{x(x+1)}=-\infty$ (7) $\lim\limits_{x\to3^+}\dfrac{x^2}{x-3}=+\infty$

(8) Given $f(x)=\begin{cases}3x-2, & x<-1\\ 4x+1, & x\ge-1\end{cases}$, find:

(a) $\lim\limits_{x\to-1^-}f(x)$ (b) $\lim\limits_{x\to-1^+}f(x)$ (c) $\lim\limits_{x\to-1}f(x)$

$=-5$ $=-3$ DNE