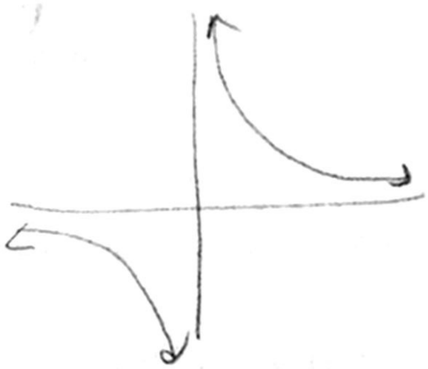
BC 1
Limits 1

Name: Owe

Continuity and One-Sided Limits

A *continuous* function has a graph which can be drawn without lifting your pencil from the paper. There are various ways a function can fail to be continuous everywhere:

(A) Sketch the graph of $y = \frac{1}{x}$.



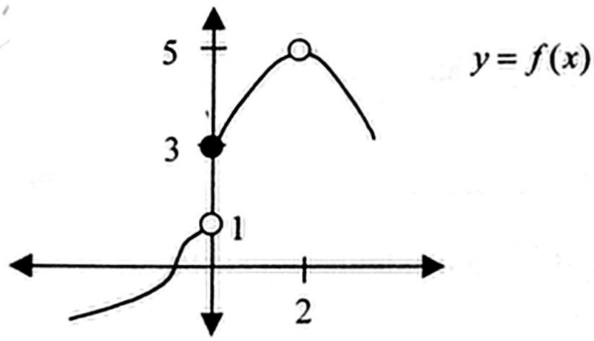
What happens to y as $x \to 0$ from the left? $y \to -\infty$

What happens to y as $x \to 0$ from the right? $y \to +\infty$

We write this as follows:

$\lim\limits_{x \to 0^-} \frac{1}{x} = -\infty$ and $\lim\limits_{x \to 0^+} \frac{1}{x} = \infty$

(B) Consider the following function $y = f(x)$:



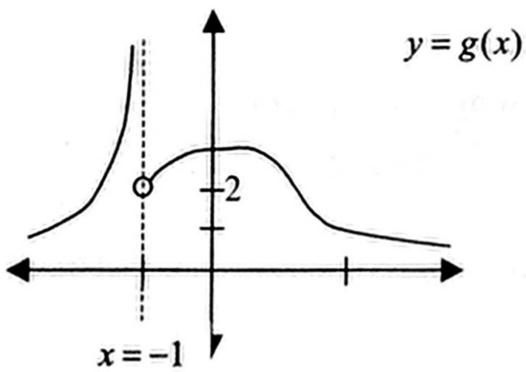
What happens to y as $x \to 0$ from the left? $y \to 1$

What happens to y as $x \to 0$ from the right? $y \to 3$

Write in limit notation:

$\lim\limits_{x \to 0^-} f(x) =$ 1 and $\lim\limits_{x \to 0^+} f(x) =$ 3

(C) Next, consider $y = g(x)$



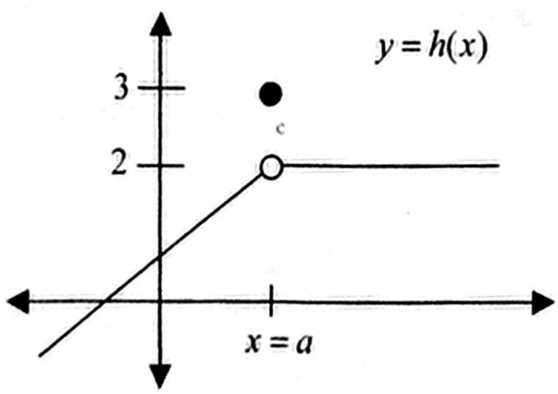
What happens to y as $x \to -1$ from the left? $y \to +\infty$

What happens to y as $x \to -1$ from the right? $y \to 2$

Write in limit notation:

$\lim\limits_{x \to -1^-} g(x) =$ $+\infty$ and $\lim\limits_{x \to -1^+} g(x) =$ 2

(D) Now, consider $y = h(x)$



What happens to y as $x \to a$ from the left? $y \to 2$

What happens to y as $x \to a$ from the right? $y \to 2$

Write in limit notation:

$\lim\limits_{x \to a^-} h(x) =$ 2 and $\lim\limits_{x \to a^+} h(x) =$ 2 more / jump? None of those on front are continuous at point in question - what goes wrong?

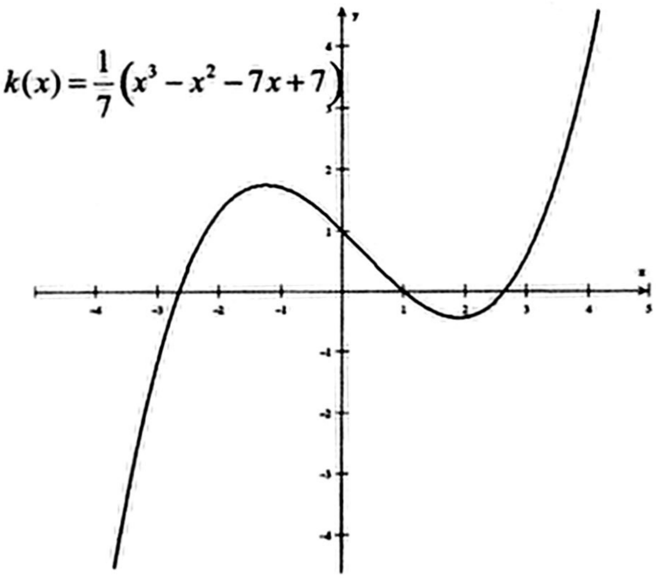
What are the conditions necessary for a function to be continuous at a point $x = a$?

$f(a)$ exists

$\lim_{x \to a} f(x)$ exists (since $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$)

$\lim_{x \to a} f(x) = f(a)$

Now, let's consider the graph of $y = k(x)$ below:



What happens to $k(x)$ as $x \to 0$ from the left?

$y \to 1$

What happens to $k(x)$ as $x \to 0$ from the right? $y \to 1$

Write in limit notation:

$\lim_{x \to 0^-} k(x) =$ __1__ and $\lim_{x \to 0^+} k(x) =$ __1__

Evaluate $k(0) =$ __1__.

Does $k(x)$ satisfy your conditions for continuity at the point $x = 0$?

yep!

For $k(x) = \frac{1}{7}\left(x^3 - x^2 - 7x + 7\right)$ above, evaluate each of the following:

$k(2) = -3/7$ $\qquad$ $k(3) = \frac{4}{7}$

What does this tell you about one of the zeros of $k(x)$? Explain. Why is continuity important here?

Since k is a poly, it is continuous. By the IVT, k must take on all values between $y = -\frac{3}{7}$ and $y = 4/7$ which includes $y = 0$. Thus, k has a zero on $[2, 3]$