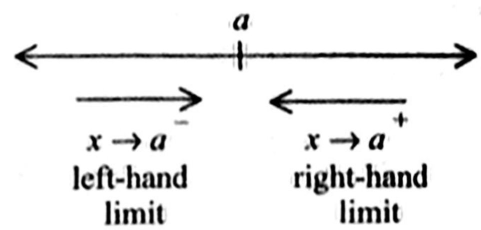
BC 1 | Name: Dra
Limits 2 | **One-Sided Limits**

So far, we have looked at different types of point discontinuities at $x = a$ by examining the behavior of the function as $x \to a$ from the left and as $x \to a$ from the right. We will now use this idea to define the limit of a function at a point $x = a$, denoted $\lim_{x \to a} f(x)$.



Consider the function $f(x) = \frac{|x|}{x}$. What is $\lim_{x \to 0} f(x)$? We would like to be able to give one, unique answer to a limit. We need some way to define, clarify, and express what happens to f at 0. We have $\lim_{x \to 0^-} \frac{|x|}{x} = -1$ and $\lim_{x \to 0^+} \frac{|x|}{x} = +1$. Since these one-sided limits are not equal, we say that the $\lim_{x \to 0} \frac{|x|}{x}$ does not exist (DNE), which means there is not one, precise answer for the limit.

$x \neq 0$

Find each of the following limits.

(1a) $\lim_{x \to 2^-} \frac{|x-2|}{x-2} = -1$ (b) $\lim_{x \to 2^+} \frac{|x-2|}{x-2} = 1$ (c) $\lim_{x \to 2} \frac{|x-2|}{x-2}$ DNE

(2a) $\lim_{x \to -1^-} [x] = -2$ (b) $\lim_{x \to -1^+} [x] = -1$ (c) $\lim_{x \to -1} [x]$ DNE

(3) Given $f(x) = \begin{cases} 2x-1, & x \le 1 \\ 3x+1, & x > 1 \end{cases}$, find:

(a) $\lim_{x \to 1^-} f(x) = 1$ (b) $\lim_{x \to 1^+} f(x) = 4$ (c) $\lim_{x \to 1} f(x)$ DNE

(4) Given $k(x) = \begin{cases} x^2-1, & x < -2 \\ x+5, & x > -2 \end{cases}$, find:

(a) $k(-2)$ DNE (b) $\lim_{x \to -2^-} k(x) = 3$

(c) $\lim_{x \to -2^+} k(x) = 3$ (d) $\lim_{x \to -2} k(x) = 3$

(5) Recall your sketch of the graph of $y = \frac{1}{x}$ on "Limits 1." Use this and your knowledge of rational functions to determine the following limits.

(a) $\lim_{x \to -4^-} \frac{1}{(x+4)^2}$ (b) $\lim_{x \to -4^+} \frac{1}{(x+4)^2}$ (c) $\lim_{x \to -4} \frac{1}{(x+4)^2}$

$= +\infty$ $+\infty$ $+\infty$

Determine each limit:

(6a) $\lim_{x \to 2^-} \frac{x-4}{x-2}$ (b) $\lim_{x \to 2^+} \frac{x-4}{x-2}$

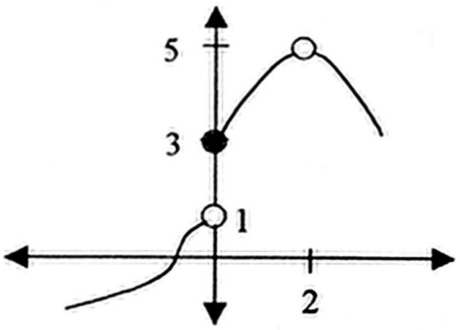
$+\infty$ $-\infty$

(7) $\lim_{x \to \frac{\pi}{2}^-} \tan x$ (8) $\lim_{x \to \frac{\pi}{2}^+} \sec x$

$+\infty$ $-\infty$

You will recall the functions $y = f(x)$ and $y = g(x)$, from "Limits 1." Use the graphs to determine each of the following limits.

(9) Here's f.



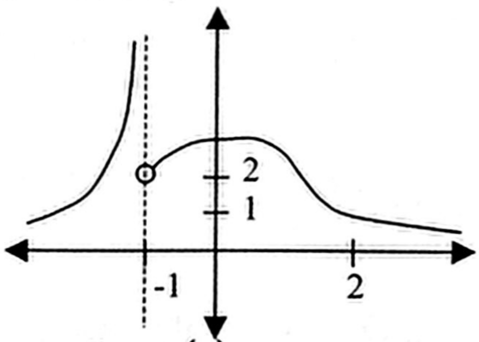
$\lim_{x \to 0^-} f(x) =$ 1 $\lim_{x \to 0^+} f(x) =$ 3

$\lim_{x \to 0} f(x) =$ DNE $f(0) =$ 3

$\lim_{x \to 2^-} f(x) =$ 5 $\lim_{x \to 2^+} f(x) =$ 5

$\lim_{x \to 2} f(x) =$ 5 $f(2) =$ DNE

(10) Here's g.



$\lim_{x \to -1^-} g(x) =$ $+\infty$ $\lim_{x \to -1^+} g(x) =$ 2

$\lim_{x \to -1} g(x) =$ DNE $g(-1) =$ DNE

$\lim_{x \to 2^-} g(x) =$ 1 $\lim_{x \to 2^+} g(x) =$ 1

$\lim_{x \to 2} g(x) =$ 1 $g(2) =$ 1